

Pioneer contributor in mathematic field  
[Enter your name here]  
MATH-199 Math/Stat Seminar  
Assignment # (The second LATEX assignment)  
[Enter the date here]

The Island of Finnøy in Nedstrand, Norway near Stavanger was the location of the birth of Niels Henrik Abel. The day of birth was the 5<sup>th</sup> of August in 1802, and died on the 6<sup>th</sup> of April 1829. In his life and even after his death, Abel was a recognized Norwegian mathematician. The contributions he made in the field of Mathematics earned him the recognition as a pioneer in multiple branches in what comprises as modern mathematics. His influence in the field was in seven years in his working life. However, in his entire life, he lived as an indigent citizen. A critical accomplishment is his single results that was adequate proof and demonstrated it is impossible to solve the generic quintic equation while they are in the form of radicals. The quintic expression refers to finding a solution for a general equation beyond the 5<sup>th</sup> degree. The problem had remained unsolved for more than 250 years before Abel's contribution.

As an innovator, Niels Abel also contributed much to elliptic functions, and additionally discovered the Abelian functions. Other contributions were in abstract algebra when his name is associated with the context of principle that proves the solution to group operations is not subjective to their order they are presented. He lived in poverty his entire life and tuberculosis brought about his demise after almost 27 years of life. The most influential connotations that define Niels Henrik Abel came from two Frenchmen, all involved in the field of Mathematics. Adrien-Marie L'engendre words could be translated into English to say, "What a head the young Norwegian has (Igarashi, Altman, Funada, & Kamiyama, 2014)." The other French Mathematician was Charles Hermite who said, "Abel has left mathematician enough to keep them busy for five hundred years (Igarashi, Altman, Funada, & Kamiyama, 2014)."

## 1. Early Life and the Good Teacher

The father to the mathematician was a lowly Lutheran minister, Søren Georg Abel, with learning degrees in philosophy and theology. The involvement of his father was evident in the

independence of the country. The parents to Niel, Soren and Anne Marie Simonsen, moved so that Soren Could serve as a minister at a parish in Gerstad, which was close to Risør town. The grandfather to Niel also served in the same church as a pastor, but Soren took the reins after the death of Hans Mathias Abel. Before serving in Gerstad, Soren and his family lived in Schleswig.

The mother to Niels was born of a merchant and enjoyed a lavish lifestyle before her marriage. Though recorded as an alcoholic, Marie was fond of the arrangement of social gatherings and balls. She, however, took minimal interest in the caring and nurturing of her children. Niels and his brothers were introduced to education by their father, with handwritten books. After the independence of Norway in 1814, Soren Abel was involved in forming the constitution. Elections also earned him a seat in representing Storting (Laudal & Piene, 2004). His position exposed Soren to the Cathedral School, which was the location for most meetings of Storting. The minister thought to send his first son Mathias, but later reconsidered in 1815 and sent Niels. Following an argument with other politicians, Soren lost his political career in 1818 and resorted to drinking only to die two years later.

The Cathedral School in Oslo was the first learning institution exposed to Niels Henrik, as from the year 1815. Niels was 13 years of age, and Mathias joined him the years later. Niels' talent in Mathematics took two years to get recognized after the arrival of a new teacher in the unit. The teacher, Bernt Michael Holmboe, gave out tests to the students and immediately recognized Niels' input. Records indicate Holmboe gave Niels private lessons after school and also managed to encourage the students to pursue the unit at an advanced level. Holmboe also advised Abel to study the original works of famous mathematicians. Some of the mathematical texts Abel used in the preparation of his research include the works of Sir Isaac Newton from the 17th century, the German Leonhard Euler from the 18th century, the Frenchman Joseph Louis Lagrange, and the German Carl Friedrich Gauss (Garding & Skau, 1994).



The teacher also proved influential in acquisition of a scholarship for Niels after the demise Soren. Niels Abel almost made him quit his studying but the influence of the teacher was adequate motivation for the Norwegian to continue in the pursuit of mathematics. Holmboe acquired assistance from the government and the Royal Frederick University to grant Abel a scholarship. A year later, Abel had graduated from the institution, but still retained ties with his teacher (Sorensen, 2010). Their bond was strong that once Abel wrote a letter to Holmboe and addresses it with  $\sqrt[3]{6064321219}$ . The result of the equation was 1823.5908275, which was the calendar date. 1823 represented the year, while the remaining decimal number, 0.5908275, and could be equated to 215.652 which rounds up to 216. The letter was thus sent on the 216<sup>th</sup> day of the year, which is the 4<sup>th</sup> of August, in the year 1823.

## 2. Binomial Theorem

The binomial theorem is concerned with the addition of two numbers when they are raised to a specified power. Euler discovered that for a non-negative integer  $n$  in a series of positive progressing numbers,  $n$  is the  $k^{\text{th}}$  term. Considering that both  $k$  and  $n$  are non-negative integers, and  $k$  is greater than  $n$ , the binomial coefficient is considered to be zero.

Binomial coefficient  $(n/k) = B(n,k)$

Therefore,

$$(a + x)^n = a^n + na^{n-1}x^1 + \binom{n}{1} \cdot \frac{n-1}{2} a^{n-2}x^2 + \binom{n}{1} \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} a^{n-3}x^3 + \dots$$

$$+ \frac{n}{1} \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \dots \frac{n-k+1}{k} a^{n-k+1}x^k$$

Niels Henrik Abel extended the discovery by Euler. He was at age 16 when he discovered that the binomial theorem is applicable to all numbers, and not restricted to just Rationals as described by Euler. The resultant theorem was named after the Norwegian mathematician, ‘Abel’s binomial theorem.’ The theorem stipulates;

$$\sum_{k=0}^m \binom{m}{k} (w + m - k)1^{(m-k-1)} (z + k)1^k = w^{-1}(z + w + m)1^m$$

### 3. Group Theory

Niels Abel invented the group theory, which is a branch of mathematics that deals with algebraic equations. The theory is highly applicable outside of mathematics, like the field of physics, making it invaluable. His invention of the theory was to provide proof that neither the roots of polynomial equations nor those of quintic equations of degree, which are greater than four, had general algebraic solutions (Podlubny, Magin, & Trymoroush, 2017). Abel then resorted to sending a letter detailing his discovery of the un-solvability of quintic equations to Carl Friedrich Gauss. However, the German took no note of the discovery but proceeded to discard the letter as worthless.

### 4. Abel’s Publications

It was in the year 1823 when Abel published his paper on integrals and functional equations. Niels was 21 at the time, and he used the paper to demonstrate the solution to an integral equation. Abel was the first in the formulation and solution. In the following year, Abel's contributions proved that it was impossible to solve, to the fifth degree, the general equation algebraically. In the hopes that the work would garner him recognition for his contributions, Abel published the works at his own expense (Ore, 1957). It was this publication that Gauss dismissed and failed to recognize the contribution by Niels that solved a famous mathematical problem. The paper proved crucial in aiding him in the acquisition of a scholarship where he traveled to France and Germany funded by the Norwegian government. Paris was the world center of mathematics in 1826, and it was then that Abel called upon the foremost mathematicians of the era. Their efforts led to the completion of a significant paper on

the theory of integrals of algebraic functions. The central result was the Abel Theorem, which formed the basis by the following methods of Abelian functions and Abelian integrals.

Abel was not successful in acquiring an appointment in Paris, and the various memoirs that he had sent to the French Academy of Sciences were lost. The excursions to Paris were not as productive as he might've hoped. He ran out of all his money and returned to Norway, but he continued his research. At the time, he had already been diagnosed with tuberculosis. Tutoring and a supplementary grant from the University of Christina helped Niels Abel to make ends meet. He, however, did not let his conditions interfere with his contributions to mathematics. There are a variety of f papers he was involved in while still in Norway, which mainly included elliptic functions and equation theory. The various contributions had earned Niels Abel fame across various mathematical centers, and multiple members from the French Academy made keen attempts to secure Abel a suitable position. After his death in 1828, it took thirteen years for the French Academy to publish his memoirs.

## 5. Elliptic Functions

While still a student, Niels Abel studied former works of elliptic functions. In 1824, Abel reported that his functions differed from the trigonometric functions because his works had two periods while the latter only had one. The fact meant Abel's functions were complex functions, and in the era, the field was relevantly in its infancy. His development in the field of elliptic functions was rapid, and he later came to learn of the contributions of Carl Gustav Jacobi in the same field, forming a competition for the discovery. However, after Abel dies, Jacobi had plenty of time to assume his contributions and continue in the development.

$$u = \int_0^x \frac{dt}{\sqrt{(1-c^2t^2)(1+e^2t^2)}}, \text{ c and e are arbitrary parameters}$$

The value of u of the integral is a function of the upper limit x of the integral. As long as  $x < 1/c$  this value will increase with increasing x and reach a maximum

$$\frac{w}{2} = \int_0^{1/c} \frac{dt}{\sqrt{(1-c^2t^2)(1+e^2t^2)}}$$

$$\varphi'(u) = \sqrt{(1 - c^2 \varphi^2(u))(1 + e^2 \varphi^2(u))}$$

$$f(u) = \sqrt{1 - c^2 \varphi^2(u)}$$

$$F(u) = \sqrt{1 + e^2 \varphi^2(u)}$$

Abel found means to extend the functions into the complex plane

$$u = \int_0^y \frac{dt}{\sqrt{(1 + c^2 t^2)(1 - e^2 t^2)}}$$

$$\frac{w'}{2} = \int_0^{1/e} \frac{dt}{\sqrt{(1 + c^2 t^2)(1 - e^2 t^2)}}$$

Abel added that  $f\left(\pm i \frac{w'}{2}\right) = \sqrt{(1 + c^2 t^2)}$  and  $f\left(\pm i \frac{w'}{2}\right) = 0$

Therefore,  $\varphi\left(u \pm i \frac{w'}{2}\right) = \pm \frac{i F(u)}{e f(u)}$

$$F\left(u \pm i \frac{w'}{2}\right) = \pm \sqrt{(c^2 + e^2 \frac{\varphi(u)}{f(u)})}, f\left(u \pm i \frac{w'}{2}\right) = \frac{\sqrt{c^2 + e^2}}{e} \frac{1}{f(u)}$$

Following the assessment that  $f\left(\pm i \frac{w'}{2}\right) = 0$ , therefore the three diversion points for the

elliptic functions are at  $\frac{w}{2} \pm i \frac{w'}{2}$  and other points related by symmetry

## 6. Abel's Theorem

Consider the following equation with a radius of convergence 1 and real coefficients  $a_k$ .

$$G(x) = \sum_{k=0}^{\infty} a_k x^k$$

If the series  $\sum_{k=0}^{\infty} a_k$  converges, it is therefore plausible to conclude  $G(x)$  is continuous from the left at  $x=1$ . Considering  $z \rightarrow 1$  borrowing from Stolz sector

$$\lim_{x \rightarrow 1^-} G(x) = \sum_{k=0}^{\infty} a_k z^k$$

Stolz Sector open disk unit

$$[1 - z] \leq M(1 - |z|)$$

$$\sum_{n>0} \frac{x^{3^n} - z^{2.3^n}}{n}$$

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